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On the wavelength of convective motions

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It has been found experimentally that under a lid of excellent heat conductivity the wavelength of convective motions on a uniformly heated plane increases with increasing supercritical Rayleigh numbers. Under an insulating lid, the wavelength at the critical Rayleigh number is larger than under a well conducting lid. When heating from below is applied rapidly the wavelength is less than in the corresponding stationary case.

1. Introduction

The wavelength is one of the most characteristic properties of the convective motions on a uniformly heated plane. The dimensionless wavelength λ is the ratio of the width of one roll to the depth of the fluid layer. According to linear theory, λ should be 1·008 at the critical Rayleigh number R_c (see Reid & Harris 1958) provided the boundaries of the fluid are rigid and of infinitely good heat conductivity. It has been shown experimentally by Koschmieder (1966), that the wavelength increases with increasingly supercritical Rayleigh numbers when the upper boundary is a thin uniformly cooled glass plate. This is not in agreement with current theories (see Platzman 1965 and Schlüter, Lortz & Busse 1965) which predict a decrease of the wavelength. The discrepancy might have been due to the glass lid, although this is not very likely, since the thermal resistance of the glass plate used in Koschmieder's experiment amounted to only 6 % of the thermal resistance of the fluid layer. But as the effective conductivity of the fluid increases after the onset of convection, the glass lid might have had undesirable consequences. It has therefore been checked, whether the wavelength increases under a lid of excellent conductivity. It is on the other hand of interest to learn whether a very poorly conducting lid will influence the wavelength. The change of wavelength with time-dependent heating has also been studied.

2. The experiments

The apparatus used in the subsequently described experiments was a slightly improved version of the set-up described in detail by Koschmieder (1966). The depth of the 1 stokes silicone oil layer (Dow Corning 200 fluid) was in this case $7{\cdot}70 \pm 0{\cdot}03$ mm while the inner diameter of the circular lucite wall containing the

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oil laterally was 200 mm. Therefore 13 concentric circular rolls should establish on the uniformly heated copper-bottom plate, according to the relation $\lambda = r/nd$ (r is the radius of the plate, d the depth of the layer) if λ is 1·008. Indeed 13 rings were observed when the fluid was covered with a 2 mm thick glass plate. The pattern covered the plate at 4·9 °C vertical temperature difference, as compared to the theoretical critical temperature difference $\Delta T_c = 4{\cdot}6$ °C which follows from $R_c = 1707$. The bottom of the lid of excellent conduction was a 8 mm thick brass plate, cooled as before. The ratio of the thermal resistances of the lid as compared to the oil layer was then 1 : 741 for pure conduction conditions, with the thermal conductivity of silicone oil $K = 3{\cdot}7 \times 10^{-4}$ cal × cm^{-1} sec^{-1} grad^{-1} and the thermal conductivity of brass $K = 0{\cdot}264$ cal × cm^{-1} sec^{-1} grad^{-1}. Since no visual observation of the motions was possible, the lid was lifted after a certain Rayleigh number was reached. Careful lifting preserves the pattern of the motions quite well. There is, furthermore, a unique determination of the wavelength made possible by the use of aluminium powder suspended in the oil to make the motions visible. At larger Rayleigh numbers, part of the powder settled under the maxima of ascending motions, here in neat rings.

The oil between top and bottom plate was now heated for over 8 h from zero vertical temperature difference to 16·8 °C or approximately $3R_c$. When the lid was lifted, there were 11 perfectly symmetrical concentric rings. The observed λ was therefore $\lambda = 1{\cdot}18 \pm 0{\cdot}05$. The innermost 11th ring was of smaller cross-section than the other rings, indicating that the proper temperature difference for the disappearance of the 12th ring had been exceeded. Under a glass lid the 12th ring just disappeared at 16·5 °C. The reduction of the number of rings from 13 to 11 under conditions which meet the assumptions of linear theory as well as possible, indicates that the increase of the wavelength with supercritical Rayleigh numbers is genuine. It should be noted that this has actually been observed earlier by Silveston (1958). The increase of the wavelength that he reports is in good agreement with the results obtained by Koschmieder (1966). Silveston's visualization technique however prohibits an effective and uniform cooling on top of the fluid, which was covered by a fairly thick glass plate. Since no cooling was applied on top of the glass plate the temperature on top of the fluid was probably not uniform. It is likely that this affected his flow pattern, which should be considered with caution. Yet this does not apply to Silveston's well known heat transfer measurements, since a carefully cooled metal lid was used in these experiments.

For the experiments with a poorly conducting plate on top of the fluid, the lid used had a 15 mm thick lucite bottom. The lucite was cooled as before while the depth of the oil was unchanged at 7·7 mm. The ratio of the thermal resistances of the lid and the resting oil layer was now around 2 : 1. Again, the fluid was slowly heated up starting with zero vertical temperature difference. In this case the copper bottom plate was covered by 12 concentric rings ($\lambda = 1{\cdot}08 \pm 0{\cdot}04$). This indicates an increase of the wavelength at the critical Rayleigh number. Increased heating then reduced the number of rings to 11. No further heating was practicable. The temperature difference across the fluid layer has, in these experiments to be determined from the measured temperature difference across the fluid layer and the lucite lid. The error introduced by this method becomes so large in the case of the thick lucite lid that no values of ΔT will be given here. However, the observed increase of the wavelength at R_c is in good agreement with a linear theory of Nield (1968). This theory describes the experimental situation here precisely and predicts an increase of the wavelength at the critical Rayleigh number and a reduction of the value of R_c when account is taken of finite conductivity of the upper boundary surface. According to Nield the wavelength should be $\lambda = 1{\cdot}11$, provided $K'/K = 1$ and $d'/d = 2$, where K' and K are the thermal conductivities of the material of the solid lid (lucite) and the fluid (silicone oil) respectively, while d' is the thickness of the lid and d the depth of the fluid.

Finally, in a series of experiments the influence of rapid heating on the onset of convection was studied. As is well-known rapid heating produces a curved time-dependent temperature profile in the resting fluid. The time unit d^2/k (thermometric conductivity $k = 1{\cdot}14 \times 10^{-3}\,\text{cm}^2\,\text{sec}^{-1}$) was 520 sec. For these experiments a lid with a cooled 2 mm thick glass plate was used, while everything else was unchanged. The procedure was as follows. The oil was first kept at rest for some hours with a small constant vertical temperature difference. The electric heating of the bottom plate was then turned on at a rate several times the amount normally necessary to reach the critical temperature difference. At an energy input 50 times the normal amount, the events were as follows: 5 min after heating started, the second circular roll formed inwards from the rim at a vertical temperature difference of 7·5 °C. At 10·8 °C, there were 3 rings. At 11 °C the fourth ring appeared, at 12 °C the fifth and sixth ring had established. All rings so far were of perfect axial symmetry. Note the very large temperature difference in comparison to the normal critical temperature difference of 4·9 °C. Inwards of the sixth ring the rings appeared very rapidly and were no longer perfectly symmetrical. Eight minutes after heating started, 15 concentric rings covered the plate at 13·5 °C temperature difference ($\lambda = 0{\cdot}87$). Note that, under stationary conditions, the number of rings at $\Delta T = 13{\cdot}5$ °C is only 12. Faster heating was not tried. The loss of perfect symmetry at the centre is most likely due to non-uniformities of the cooling. The experiment just described indicates that the wavelength *decreases* if the heating is time-dependent. This is in agreement with an observation made recently with an oil layer under an air-surface (Koschmieder 1967). The apparent increase of the critical temperature was to be expected. Suppose one applies the normal ΔT_c suddenly on the oil layer and approximates the resulting curved vertical temperature distribution by two straight sections, the upper subcritical and the lower supercritical for the normal ΔT_c. Consider now, that the critical temperature difference is inversely proportional to the third power of the depth of the layer, which will still be approximately true with quasi-stationary conditions. Therefore the proper critical temperature difference will not be exceeded either for the upper region or for the lower region, even if ΔT_c for the whole layer is applied only to the lower region. In other words, the onset of convection with time-dependent heating requires a temperature difference larger than the normal critical difference.

Time-dependent heating has been studied theoretically by Goldstein (1959) and Lick (1965). The latter work is restricted to a layer which is narrow in comparison with the overall depth of the fluid. Goldstein's assumptions match the situation in the aforementioned experiment. He predicts a marked increase of the critical Rayleigh number with rapid heating and a slight decrease of the wavelength. According to his calculations not more than 14 rings should have established at around $3R_c$, while actually 15 rings appeared. Finally, Foster (1965) has observed a decrease of the wavelength at very high Rayleigh numbers, when the fluid was cooled from above and a time-dependent temperature distribution developed.

3. Conclusions

As is well known, the wavelength of convective motions at the onset of convection is the result of a balance between the kinetic energy dissipated by viscosity and the internal energy released by buoyancy. The wavelength varies as this balance is altered by either rapid heating or poor conduction on top of the fluid. More energy must be released in the case of rapid heating, while less energy can be released when conduction on top of the fluid is poor. As we observe apparently more energy is released by more cells (rapid heating) and less energy by fewer cells (poor conduction). Suppose that in both cases there is still a balance between dissipation and energy release. Then our observation indicates that dissipation is proportional to the number of cells or increases with decreasing wavelength as we anticipated. But how do we explain that with increasingly supercritical Rayleigh numbers the number of cells decreases? Certainly more energy must be released than at the critical temperature difference, since the applied temperature difference is larger. On the other hand dissipation increases as the velocities of the motions increase rapidly. Apparently the increase in dissipation is dominant and shifts the balance of energy release and dissipation to larger wavelengths. In other words, the fluid reduces the number of rolls to offset the increase in dissipation in a supercritical cell, as compared to the dissipation in a just critical cell.

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